

# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

M.A./M.SC. SECOND SEMESTER EXAMINATION, MAY 2012

FIRST YEAR

MATHEMATICS

Date : 30/05/2012

Time : 11 am – 2 pm

Paper : VIII

Full Marks : 60

Answer **any three** questions :

[3×20]

1. a) Explain how the product of a generalized function  $f \in \mathcal{D}'$  by an infinitely differentiable function  $\alpha(x)$  is defined? Show that  $\alpha(x)\delta(x) = \alpha(0)\delta(x)$ .

- b) Find Green's function of the problem

$$\frac{d^2u}{dx^2} + \frac{2}{x} \frac{du}{dx} - \frac{6}{x^2} = \begin{cases} x & , 0 \leq x \leq 1 \\ \frac{1}{x^2} & , x > 1 \end{cases}$$

with boundary conditions :  $u \rightarrow 0$  as  $x \rightarrow 0$  and as  $x \rightarrow \infty$ . Hence find the solution of the problem.

2. a) Prove the Plemelj's formula,  $\lim_{\epsilon \rightarrow 0} \frac{1}{x+i\epsilon} = -\pi i \delta(x) + P \frac{1}{x}$ .

- b) Find the solution of the integral equation,

$$\phi(x) = \frac{1}{2} \int_{-1}^1 (xt + x^2 t^2) \phi(t) dt + x^2,$$

by the method of resolvent kernel.

3. a) By the use of Fourier transform, show that the solution of the equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, -\infty < x < \infty, y > 0, \text{ satisfying the boundary conditions,}$$

$u(x, 0) = f(x), -\infty < x < \infty, u(x, y) \rightarrow 0$  as  $y \rightarrow \infty$  and  $u(x, y), u_x(x, y) \rightarrow 0$ , as  $|x| \rightarrow \infty$ , is

$$u(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi)}{(x-\xi)^2 + y^2} d\xi.$$

- b) State and prove the convolution theorem of Laplace transform.

4. a) State the convolution theorem of Fourier transform. If  $f(x) = H[\psi(x)]$ , show that  $\psi(x) = -H[f(x)]$ , where  $H$  is the Hilbert transform operator.

- b) Show that the integral equation

$$\phi(x) = f(x) + \frac{1}{\pi} \int_0^{2\pi} \sin(x+t) \phi(t) dt$$

possesses no solution for  $f(x) = x$ , but it possesses infinitely many solutions when  $f(x) = 1$ .

5. a) Find by evaluating the inversion integral the function whose Laplace transform is  $\frac{1}{(p^2+1)^4}$ .

- b) Show that the solution of the integral equation,

$$\int_0^x \frac{\phi(t)}{(x-t)^\alpha} dt = f(x), 0 < \alpha < 1 \text{ is } \phi(x) = \frac{\sin \alpha \pi}{\pi} \frac{d}{dx} \int_0^x f(t)(x-t)^{\alpha-1} dt.$$

